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Thick Accretion Disks:

So far we have considered various accretion scenarios. There are situations with low accretion rates and very low specific angular momentum (spherical accretion). There are scenarios in which the infalling angular momentum cannot be ignored. The accreting gas in this case forms a rotating disk. Thin disks emit the released gravitational energy efficiently via radiation, and this is the reason why they are geometrically thin.

There is another accretion scenario that is likely to apply to AGN's and QSO's. In AGN's the accretion disk may be geometrically thick for at least two reasons. It may radiate inefficiently, in which case the orbiting plasma retains much of the dissipated gravitional energy. Also, an AGN disk may be accreting at a super-Eddington rate, in which case the transfer of radiation through the optically thick medium cannot keep up with the heating rate. In both the cases, the disk is prevented from collapsing to the equatorial plane.

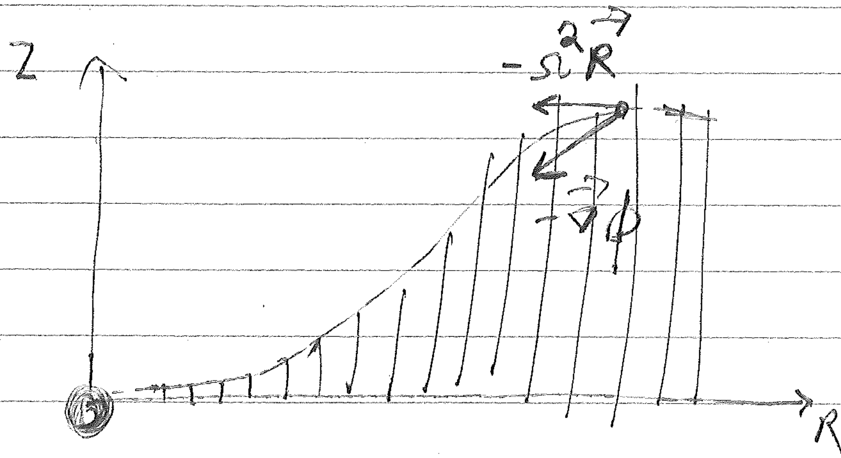
As a side note, we learned at the very beginning that if the aceretion rate exceeds the Eddington limit, the accretion will be heavily suppressed due to outward radiation pressu However, this is strictly true for a spherically symmetric geometry. For a thick disk configuration, as we will see, the Eddington limit may be circumvented, thus allowing emission of radiation at a more efficient rate.

We start with the structure of thick disks. In these systems the vertical structure becomes important. It is easy to see that pressure gradient plays an important role in dynamics. For example, consider an axi-symmetric configuration, where:

$v_R \simeq 0$, $v_\phi \simeq R\Omega$, $v_z \simeq 0$

Then:

$\frac{1}{\rho}\vec{\nabla}P = -\vec{\nabla}\Phi + \Omega^2\vec{R}$



As seen from the figure, one cannot get rotation about the z-axis without the $\vec{\nabla}P$ term. This implies that the rotation law will be non-Keplerian, where $\Omega = \Omega(R,z) \neq \Omega_K(R)$.

The surface of the disk, represented by the relation $z = z_s(R)$, is an isobaric surface. Therefore $\vec{\nabla}P|_s = 0$, resulting in:

$-\vec{\nabla}\Phi|_s + \Omega^2\vec{R}|_s = 0$

Or:

$$(\Omega^2 R)_s = \left(\frac{\partial \phi}{\partial z}\right)_s \frac{\partial z_s}{\partial R_s} + \left(\frac{\partial \phi}{\partial R}\right)_s$$

To illustrate some important differences arising for a thick disk geometry, let us consider a simple situation where the disk's boundary is a cone, in which case:

$$z_s(R) = \pm (\tan\alpha) R$$

In this case:

$$\phi = \frac{-GM}{(R^2+z^2)^{1/2}} \Rightarrow \frac{\partial \phi}{\partial z} = \frac{GMz}{(R^2+z^2)^{3/2}} \Rightarrow \frac{\partial \phi}{\partial z}\Big|_s = \frac{GM \tan\alpha \cos^3\alpha}{R^2}$$

$$\frac{\partial \phi}{\partial R} = \frac{GMR}{(R^2+z^2)^{3/2}} \Rightarrow \frac{\partial \phi}{\partial R}\Big|_s = \frac{GM \cos^3\alpha}{R^2}$$

Putting things together, we find:

$$(\Omega^2 R)_s = \frac{GM \tan^2\alpha \cos^3\alpha}{R^2} + \frac{GM \cos^3\alpha}{R^2} = \frac{GM \cos\alpha}{R^2}$$

Thus:

$$\Omega^2(R) = \frac{GM \cos\alpha}{R^3}$$

It turns out that this particular type of disk is Keplerian, but with a reduced effective mass $MG\,s\alpha$. The reduction is due to the opposing effect of pressure gradient from the gas. Note that $\Omega \to \Omega_K$ as $\alpha \to 0$, which is expected in the thin disk limit.

Let us now see how the disk shape can affect the luminosity. Recall that the radiative flux is given by;

$$\vec{F}_{rad} = \frac{-c}{k\rho}\vec{\nabla} P \qquad (k\text{; opacity})$$

It can be written as;

$$\vec{F}_{rad} = \frac{c}{k}\vec{\nabla}\phi - \frac{c}{k}\Omega^2\vec{R}$$

The luminosity is therefore going to be;

$$L = \frac{c}{k}\oint \vec{\nabla}\phi \cdot d\vec{s} - \frac{c}{k}\oint \Omega^2\vec{R}\cdot d\vec{s} \underset{\text{Gauss's law used}}{=} \frac{c}{k}\int_V \nabla^2\phi \, dV - \frac{c}{k}\int_V \vec{\nabla}\cdot(\Omega^2\vec{R})\, d$$

Poisson's equation for the gravitational field states tha

$\nabla^2 \varphi = 4\pi G \rho$. Thus:

$$L = \frac{c}{\kappa}\int_V 4\pi G \rho \, dV - \frac{c}{\kappa}\int_V \vec{\nabla}\cdot(\Omega^2 \vec{R})\, dV = \frac{4\pi c G M}{\kappa} - \frac{c}{\kappa}\int_V \vec{\nabla}\cdot(\Omega^2\vec{R})\, dV$$

Note that the first term on the right hand side is just the Eddington luminosity for mass M. To see the effect of the second term, we write:

$$\vec{\nabla}\cdot(\Omega^2 \vec{R}) = \frac{1}{R}\frac{d}{dR}(\Omega^2 R^2) = 2R\Omega\frac{\partial \Omega}{\partial R} + 2\Omega^2$$

Now lets use the expression we obtained for Ω in the case of simplified geometry:

$$\Omega^2 = \frac{GM\cos\alpha}{R^3} \Rightarrow \vec{\nabla}\cdot(\Omega^2 R) = -\Omega^2$$

Thus:

$$L = L_{edd} + \frac{cGM\cos\alpha}{\kappa}\int_V \frac{1}{R^3}\, dV \quad , \quad dV = 4\pi R^2 \tan\alpha \, dR$$

Finally, we arrive at the following expression:

$$L = L_{edd}\left[1 + \sin\alpha \ln\left(\frac{R_2}{R_1}\right)\right]$$

Here R_1 and R_2 are the inner and outer radii of the funnel region in the thick disk respectively. The important thing is that $L > L_{edd}$. For example, with $\alpha = 45^\circ$ and $R_o = 100 R_{ms}$, we have $L \sim 4 L_{edd}$. Therefore, an AGN can emit at over 4 times the Eddington limit. We notice that the vertical extension in these structures is a direct result of the gravitational dissipation associated with such large accretion rates.